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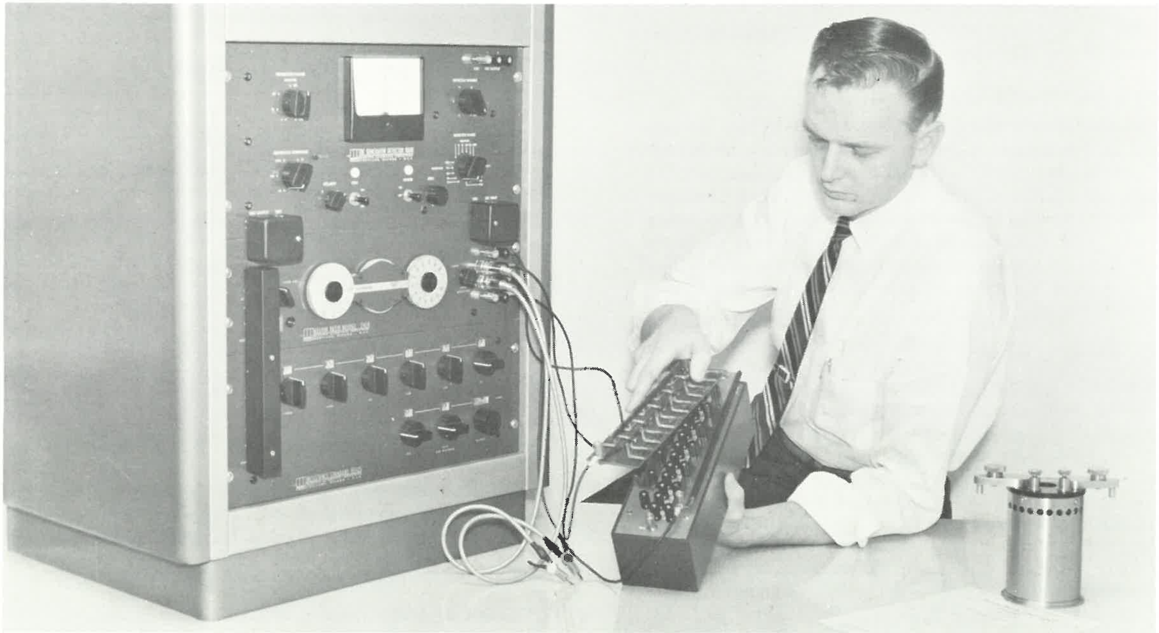

PRECISION RESISTANCE MEASUREMENT—PART II

In Precision Resistance Measurement, Part I, we discussed the use of four-terminal resistance measurements for minimizing the effects of lead and contact resistances, and the use of three-terminal resistance measurements for reducing the effects of stray shunt resistances. In Part II, we shall show how a resistor of any value can be calibrated with predictable accuracy, relative to a single certified resistance standard.

The accuracy of direct bridge readings is limited by the stability of the many bridge resistors and other factors. Comparison measurements using the same bridge find the value of one unknown resistor relative to a standard of approximately equal value to much greater accuracy. Transfer standards or build-up resistors can then be used to transfer resistance value in precise ratios. For example: 10 equal resistors can be connected in series, in parallel, or in series-parallel combinations. One of the combinations can be calibrated and the values of other combinations can then be calculated. This technique provides a method for "transferring" resistance values.

First, however, we must obtain a resistance standard of known accuracy from which to transfer. Extremely stable and accurate standard resistors are available. They can be transported to the National Bureau of Standards for certification. One or more of these units and their certificate values provide a reference standard of resistance for a standards laboratory. This reference standard is used to calibrate working standards in the laboratory.

It is easy to make precision resistance measurements relative to a certified standard over a wide range of resistance values with a set of ESI Model SR-1010 Resistance Transfer Standards and an ESI Model 241 or Model 242 Resistance Measuring System. The transfer standards and the bridge have been designed for four-terminal connections so that low resistance values can be measured with high accuracy. The SR-1010 boxes have special resistor junctions which permit full accuracy even when low resistance units are connected in parallel.

What Is an Ohm?

Before we can get very far with a discussion of precision measurements, we need to reach an agreement as to what we mean by "one ohm". There are several meanings and at the one ppm level the resulting values can be different. We formerly used the "International Ohm", but we now use the "Legal Absolute Ohm" which is the standard maintained by the National Bureau of Standards.

International Ohm

In 1894, the United States Congress defined the ohm as "the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass of a constant cross-sectional area, and of the length of 106.3 cm". This was the "International Ohm" and many of the older standard resistors were made using this unit. These resistors are about 0.05% high in terms of the present standard.

Absolute Ohm

Since July 21, 1950, the ohm in the United States has been defined as "—one thousand million units of resistance of the centimeter-gram-second system of electromagnetic units—". This is the "absolute" ohm proposed by the International Committee on Weights and Measures.

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Legal Absolute Ohm (NBS Standard)

Congress also determined that the legal value of the ohm "—shall be—represented by, or derived from, national reference standards maintained by the Department of Commerce—". This means that the resistance standard maintained by the National Bureau of Standards is exactly correct, legally, although it may vary by a few ppm from the true value of the "absolute ohm". The standards of several countries are compared frequently and the NBS standard is in agreement with the resulting value determined by the International Bureau of Weights and Measures. When we have a resistor certified, the measured value on the certificate is in "legal absolute ohms" and the accuracy value refers only to measurement accuracy and expected stability for one year. No figure is given for the accuracy of the NBS standard in "absolute ohms". In most discussions, the NBS values are called "absolute ohms" because the Standard at NBS is in agreement with the internationally accepted value of the absolute ohm.

Reference Standard Ohm

We need a reference standard of resistance in our laboratory which is traceable to the National Bureau of Standards. To get this, we send stable Standard Resistors to NBS for annual certification. We assume that these resistors maintain their certified value within their certified accuracy for the year. The reference standard may be one such resistor or it may be a value derived from several units. The reference standard is chosen as the value in which we have the most confidence.

Precision Measurement

Accurate ratio techniques can be used to transfer from one resistance value to another so we actually need only one reference standard value to keep our resistance measurements in line. The accuracy of the ratio measuring equipment can be checked in any reasonably well-equipped laboratory so that NBS certification of such equipment is not necessary. Furthermore, the accuracy of measurements made with ratio measuring equipment far exceeds the long-term stability of the equipment. Therefore, a certificate stating the expected stability of an instrument would give an unreasonably pessimistic indication of its ability.

Build-up Resistors

Resistors are calibrated relative to standards of a different value by using a comparison bridge, or ratio set, and transfer, or build-up resistors. In 1940, Dr. Wenner wrote in the "Journal of Research of the National Bureau of Standards" that "Presumably Lord Rayleigh was the first to use the same coils connected alternately in parallel and in series in building up from unit standards." Let's find out how these build-up resistors are used and what measurement accuracies they will yield.

Ten resistors, each of resistance R, can be connected in series to give 10 R, in parallel to give R/10, and 9 of them can be connected in series-parallel to give R with greater stability and higher power dissipation than a single resistor. The major advantage of using build-up resistors is that the accuracy may be almost identical at all three resistance values. To see why this is true, let us take ten approximately equal resistors of nominal value R, but each with an actual value R_n which deviates by Δ_n from R.

$$R_n \equiv R\,(1+\Delta_n)$$

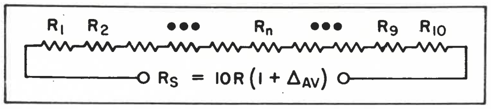
where
R_n – Resistance of nth resistor
R – Nominal resistance of each resistor
Δ_n – Deviation of R_n from R in proportional parts

We will see that for reasonably well matched resistors both the series and parallel combinations will deviate from their nominal value by the average deviation Δ_{AV} of the deviations Δ_n of the individual resistors.

$$\Delta_{AV} \equiv \frac{1}{10}\sum_{n=1}^{10}\Delta_n$$

where
Δ_{AV} – Average of the deviations Δ_n

Series Connection of 10 Resistors



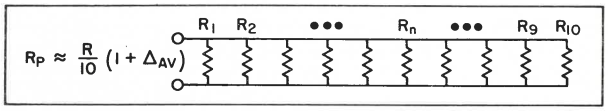
When we connect the ten resistors R_n in series, we get a resistance R_S which is approximately 10 times the resistance of each.

$$R_S = \sum_{n=1}^{10} R_n = \sum_{n=1}^{10} R\,(1+\Delta_n) = 10R\left(1+\frac{1}{10}\sum_{n=1}^{10}\Delta_n\right)$$

$$R_S = 10R\,(1+\Delta_{AV})$$

where
R_S – Resistance of 10 resistors in series
R – Nominal resistance of each resistor
Δ_{AV} – Average deviation of the 10 resistors

Parallel Connection of 10 Resistors



If we connect the same ten resistors R_n in parallel, we will get a resistance R_P:

$$R_P = \frac{1}{\sum_{n=1}^{10}\frac{1}{R_n}}$$

where R_P – Resistance of 10 resistors in parallel

We can use Taylors' series to find $1/R_n$

$$\frac{1}{R_n} = \frac{1}{R\,(1+\Delta_n)} = \frac{1}{R}\,(1-\Delta_n+\Delta_n^2+\bullet\bullet\bullet)$$

Then R becomes

$$R_P = \frac{1}{\sum_{n=1}^{10}\frac{1}{R}(1-\Delta_n+\Delta_n^2+\bullet\bullet\bullet)} = \frac{R}{10}\left[\frac{1}{1+\frac{1}{10}\sum_{n=1}^{10}(-\Delta_n+\Delta_n^2+\bullet\bullet\bullet)}\right]$$

A second application of Taylors' series gives:

$$R_P = \frac{R}{10}\left[1+\frac{1}{10}\sum_{n=1}^{10}\Delta_n-\frac{1}{10}\sum_{n=1}^{10}\Delta_n^2+\left(\frac{1}{10}\sum_{n=1}^{10}\Delta_n\right)^2+\bullet\bullet\bullet\right]$$

The second order terms cannot exceed the square of the maximum value Δ_{MAX} of Δ_n so R_P becomes:

$$R_P = \frac{R}{10}\left[1+\Delta_{AV}+\begin{cases}+0\\-(\Delta_{MAX})^2\end{cases}+\bullet\bullet\bullet\right]$$

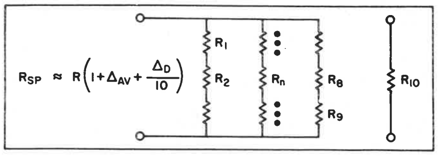
where Δ_{MAX} – Magnitude of largest deviation Δ_n

If Δ_{MAX} does not exceed 100 ppm then $(\Delta_{MAX})^2$ cannot exceed 0.01 ppm. This is negligible. The contribution of the higher order terms is even less. So if we use precision resistors we find the R_P deviates from its expected value by Δ_{AV} just as R_S did.

$$R_P \approx \frac{R}{10}\,(1+\Delta_{AV})$$

The resistors in the ESI Model SR-1010 Transfer Standard are matched so that this expression for R_P is precise enough for even the most accurate measurements.

Series Parallel Connection of 9 Resistors


$R_{SP} \approx R\left(1+\Delta_{AV}+\frac{\Delta_D}{10}\right)$
R_1
R_2
R_n
R_8
R_9
R_{10}

If nine of our ten resistors are connected in series-parallel as shown we will get a resistance R_{SP} which is approximately equal to the nominal resistance of each resistor. An analysis of this circuit (ESI Engineering Bulletin No. 25) shows that the resistance of the first nine resistors connected in series-parallel is:

$$R_{SP} = R\left[1 + \Delta_{AV} + \frac{\Delta_D}{10} + \begin{Bmatrix} +0 \\ -(\Delta_{MAX})^2 \end{Bmatrix} + \cdots\right]$$

where

R_{SP} – Resistance of 9 of 10 resistors R_i connected in Series-Parallel

Δ_D – Deviation of R_{SP} from the tenth resistor R_{10}

$$R_{SP} \equiv R_{10}\,(1 + \Delta_D)$$

Second and higher order terms can again be neglected because they are so small when we use accurate resistors. For precision measurement purposes, we can consider that R_{SP} is given by

$$R_{SP} \approx R\left(1 + \Delta_{AV} + \frac{\Delta_D}{10}\right)$$

Calibrating Working Standards

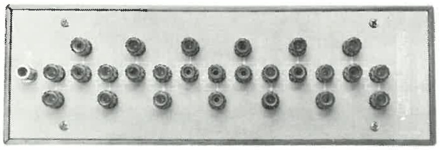
If we have a reference standard with a value of ten R or one-tenth R, we can connect ten transfer standard resistors in series or parallel and measure $\triangle_{AV}$. We can compare R_{SP} to R_{10} and measure $\triangle_D$. Then we can calculate R_{SP}. If our standard has a value of R we can compare it with R_{SP} and measure $(\triangle_{AV} + \triangle_D/10)$. We can measure $\triangle_D$. and calculate $\triangle_{AV}$. The result is 3 calibrated resistance measurement values, 0.1R, R and 10R.

Transfer measurements can be made quickly and easily. They provide the ultimate in measurement accuracy. Transfers can be made in steps other than decade jumps, too. If each resistor in each SR-1010 box is calibrated, the value of any combination can be calculated, relative to the reference standard, and used for accurate comparison measurements at intermediate resistance values. The series connection of different numbers of resistors is particularly useful for calibrating the resistance of a decade.

Four-Terminal Transfer Measurements

Needless to say, at low resistance levels, the accuracies of a few ppm which we have been discussing cannot be achieved with conventional connection methods. Contact variations of as little as 100 microhms can upset the resistance of a 10 ohm per step box enough to destroy the desired accuracy. Fortunately, four-terminal measurement techniques can be used for these measurements. With four-terminal measurements, the usual values of lead and conection resistance will have no appreciable effect.

Four-Terminal Transfer Standard



A very useful transfer standard can be built by connecting several equal resistors permanently in series. If two terminals are brought out from each resistor junction, we can make four-terminal measurements of each individual resistor and of any number of resistors connected in series. We can also make four-terminal parallel connections with high accuracy.

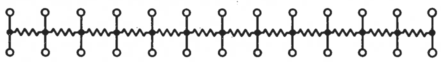


Figure 1. Transfer standard with two terminals at each junction.

Resistor Junctions

When two terminals are connected to a junction between two resistors, considerable care must be taken. It is no easy matter to make both terminals connect to a single point. The terminals of the SR-1010 boxes have been designed so that four-terminal connections and measurements can be made with part per million accuracy even with the low resistance units. The junction connections can be easily checked. Four leads enter each junction. Current between any two leads should not produce any measurable voltage between the other two leads.

Four-Terminal Series Connection

A four-terminal measurement can be made of each resistor in Figure 1 using the terminal pairs brought out from the junctions. The resistance of several resistors permanently connected in series is the sum of the individually measured resistances. This series combination can be measured, or used as a four-terminal standard, by connecting to the terminal pairs at each end.

Four-Terminal Parallel Connections

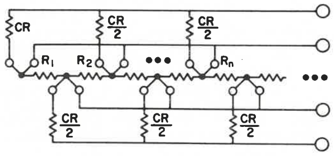


Figure 2. Four-terminal parallel connection.

The parallel connection of four-terminal resistors is somewhat more complicated. It has been a problem in the design of low resistance shunts for many years. In 1912, Dr. Wenner discussed the problem in the "Journal of the National Bureau of Standards". Recently, the shunt-connecting technique has been adapted for connecting low resistance build-up resistors to make precision measurements. Mr. B. V. Hammon described such a unit in "The Journal of Scientific Instruments" in December, 1954.

A detailed analysis of this method for connecting many four-terminal resistors in parallel is quite involved. Fortunately, we can get some appreciation for the problems—and find ways of avoiding them—by analyzing a more simple circuit. Two four-terminal resistors can be connected in parallel and the resulting four-terminal resistance can be analyzed by the judicious application of Kirchoff's laws. (Send for ESI Engineering Bulletin No. 25). The analysis shows us two ways to reduce the error at each end. If one set of connection resistors at each end can be reduced to zero, we will have no problem, so the closer to zero they become, relative to the resistor being measured, the less our problem will be. Also, there will be no error if one set of the connection resistors on each end are in the same proportion as the resistors being measured. Thus, we should be able to make an accurate parallel connection if we make one set of resistors small and the other set in approximately the correct ratio. If the resistance of the low resistance connections is less than M times the resistance of the resistors being connected in parallel, and connection resistances do not deviate from each other by more than $\triangle$, then the measured parallel resistance will have a maximum deviation $\triangle_M$ from its calculated value, where:

$$\Delta_M = 2M\Delta$$

where:

M – Ratio of low connection resistance to resistance being paralleled

Δ – Unbalance of high connection resistances in proportional parts

For example: We can connect several 10 ohm resistors in parallel and measure the resistance of the combination to better than ±2 ppm accuracy if:

1. The low resistance connections are less than 10 milliohms ($M < 0.001$).
2. The high resistance connections are matched to better than 0.1% ($\triangle < 0.001$).

Four-Terminal Series-Parallel Connection

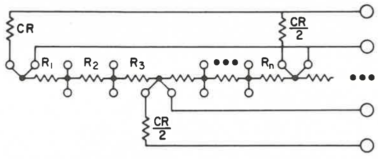


Figure 3. Four-terminal series parallel connection.

When nine resistors are four-terminal connected in series-parallel, as in Figure 3, the deviation due to connection is again:

$$\Delta_M = 2M\Delta$$

but if we use the same connection resistances as before, this M is one-third of the previous one, so the accuracy is improved by a factor of three.

Adjustable Connection Resistors

If connection fixtures with low enough values of M and Δ are not available, an adjustment technique can be used for paralleling four-terminal resistors. This technique will work even when the resistors R_n are decades different in value! We have seen that accurate measurements can be used if the connecting resistors are in exactly the same proportion as the resistors being connected in parallel. It is possible to *adjust* connecting resistors to do this. The adjustment circuit is shown in Figure 4.

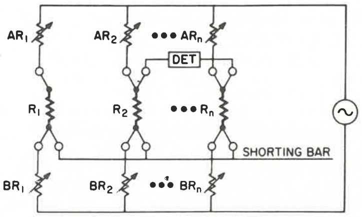


Figure 4. Adjusting parallel connection of four-terminal resistors.

Connect the resistors to a power source which will not cause enough heating to shift their value, but will produce at least the current that will be used in a bridge measurement. Connect the bottom terminals with a shorting bar. Place a sensitive detector, (at least as sensitive as the bridge detector to be used), alternately between pairs of the upper terminals as shown. Adjust the upper variable resistors until the detector reads a null between every pair of terminals. Disconnect the shorting bar from the lower terminals and connect it to the upper terminals. Move the detector to the lower terminals and adjust the lower resistors for zero voltage between terminals. Disconnect the upper shorting bar and check both the upper and lower terminal pairs for null. Now both ends of all of the resistors to be measured are at the same potential. Put both shorting bars on and connect the unit to the bridge, as shown in Figure 5. The bridge will measure the parallel resistance of the resistors accurately.

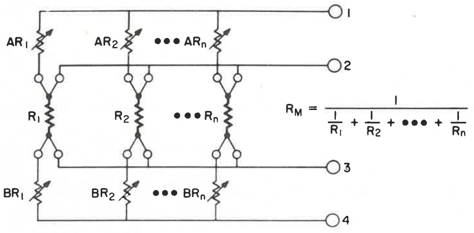

1
2
3
4
AR₁
AR₂
••• ARₙ
R₁
R₂
•••Rₙ
BR₁
BR₂
••• BRₙ
$R_M = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$

Figure 5. Measuring parallel connected four-terminal resistors.

Measurement Systems

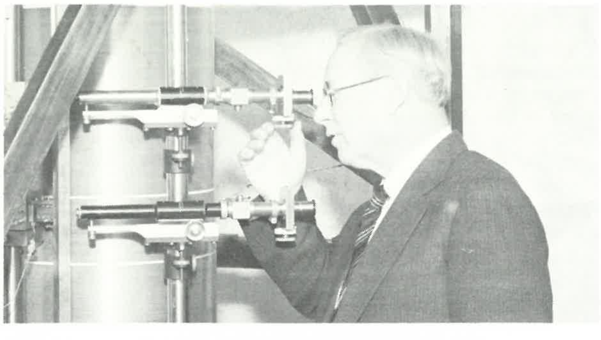
The ESI Model 241 Resistance Comparison System, the ESI Model 242 Resistance Measuring System, and the SR-1010 Resistance Transfer Standards have been designed to take full advantage of comparison techniques for precision measurements of resistance and ratio. These systems feature precision stability and ease of operation. With the techniques which have been discussed in these first three issues of "Design Ideas", they can be used to calibrate dividers within a fraction of a ppm and to measure resistance to absolute accuracy of a few ppm.

Our Apoligies to Lord Kelvin

Mr. Chester Peterson of the National Bureau of Standards pointed out that in "Design Ideas" Vol. I, No. 2, we should have spelled Lord Kelvin's name THOMSON.

Also in the first printing of No. 2, the equation in Figure 5, should have read

$$X = S\left(\frac{A}{B}\right) + M\left(\frac{A}{B} - \frac{N}{M}\right)$$



Dr. Frank Wenner

Our last biographical sketch was concerned with the great English Scientist, Sir William Thomson, Baron Kelvin of Largs. The late Dr. Frank Wenner, formerly of the United States National Bureau of Standards, is a logical choice to span the history of precision measurements from Lord Kelvin's day to very nearly the present.

Every great leader in the scientific field has been characterized by a combination, to an uncommon degree, of mechanical genius and theoretical brilliance. These, plus endless hours of hard work, have never failed to produce outstanding achievements. Born in 1873 in Iowa, Dr. Wenner early showed his mechanical genius by assembling and repairing some of the first mechanical binders and threshing machines. A bent for long, hard work was a necessity on his father's farm. Theoretical brilliance had to wait, since farm work made possible only seasonal attendance at school. By making up about two years of Latin, he was allowed to enter Knox College at Galesburg, Illinois in 1895, where his work in physics and mathematics earned him a student assistantship and a membership in the local honor society. After taking his degree in 1899, he was associated with the teaching staffs of Knox College, University of Wisconsin, Iowa State College, and the University of Pennsylvania. In 1907, while working on his doctorate at the University of Pennsylvania, Dr. Wenner joined the Scientific staff of the National Bureau of Standards. He remained at the Bureau until his retirement in 1953.

Like Lord Kelvin, Dr. Wenner made significant contributions in many fields. For his design of the Wenner Seismometer, the Franklin Institute presented him the John Price Wetherill medal. He was also an American delegate to the Fourth General Assembly of the International Geodetic and Geophysical Union in Stockholm. It was at about this time, that he invented the Wenner Potentiometer. Dr. Wenner originated several methods for measuring electrical resistances in fundamental units. One of these methods has been used very successfully at the National Bureau of Standards, and the results were reported to the International Bureau of Weights and Measures in 1938.

His main field of interest was the Bureau's Resistance Measurement Section, where his over thirty years of service produced a host of important scientific publications, many of them fundamental in nature. One of the most important of these was his paper, "The Four-Terminal Conductor and the Thomson Bridge", written in 1912. This paper is a classic discussion of four-terminal paralleling of resistance elements.

Litho in U.S.A.